A contextual Choquet integral-based preference learning model considering both criteria interactions and the compromise effects of decision-makers

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Abstract

Preference learning has been widely employed to predict decision-makers' preferences from historical information. This study develops a preference learning model for multiple criteria decision analysis where the decision-maker is supposed to be bounded rational and criteria are not completely independent of each other. The contextual Choquet integral is used as the aggregation function to address criteria interactions. The robust-ordinal-regression (ROR) technique is then applied to learn the preferences of decision-makers from the given preference data and provide robust decision recommendations. The proposed approach is illustrated by a numerical study concerning sustainable product evaluation.

Keywords: Multiple criteria analysis; preference learning; compromise effect; interactive criteria; robust ordinal regression

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1. Introduction

Selecting the best alternative from a finite set of candidate alternatives evaluated against multiple criteria is an important multiple criteria decision making (MCDM) problem (Greco et al. 2016; Dhurkari 2022). For such problems, a decision-maker chooses the item with the largest utility. Expected utility theory, the most popular methodology for MCDM, explains the behavior of decision-makers under the assumption of rational choice: people always maximize expected utilities (Dyer et al. 1992). However, the rational choice theory has been challenged continuously. Numerous empirical studies on human cognition have documented the apparent irrational behaviors based on different assumptions of conceptual and perceptual processing (Howes et al. 2016). Among those behavioral anomalies, *Compromise effect* (or saying, extremeness aversion) is a widely observed form of bounded rational behaviors (Guo 2016). There has been a great deal of research on developing and deploying methods to capture such irrational preferences in the fields of, for example, consumer behavior analysis (Kivetz et al. 2004) and marketing strategy (Guo 2016). These methods combined decision analysis and behavioral economics to capture compromise effect and identify the most preferred alternative or yield predications. These theories may give rise to collapse of using traditional MCDM methods which do not consider decision-makers' irrational behaviors.

MCDM applications commonly utilized the additive preference model (Keeney & Raiffa 1976; Greco et al. 2008; Reimann et al. 2017). In this model, each alternative is first evaluated among a set of criteria. The overall value is then obtained as the sum of multiple values of the alternative over multiple criteria. A concern with the additive preference learning model is that, it is not particularly well-suited for handling interactions between criteria.

Consequently, the ordered weighted averaging (OWA) operator has been proposed to capture the permutation of all criteria values (Yager 1998; Reimann et al. 2017). It is worth highlighting that, even though Yager (1998) utilized aggregation functions to capture multicriteria preferences at alternatives, these aggregation functions were not directly used by the additive preference model. Further, results of Grabisch (1997a) showed that the OWA operator does not identify the positive (or negative) interactions among interactive criteria. Hence, although weighted value functions provide information about interactions, the additive preference model lacks a solid foundation to accommodate these interactions.

We address these gaps in the existing literature by developing a contextual Choquet integral for MCDM with interactive criteria. Specifically, we show that with available information about interactions among multicriteria, these preferences can be represented with a concave aggregation function. Importantly, this aggregation function makes use of a fuzzy measure constructed over the whole set of criteria. It is able to accommodate the interactions that the additive value function fails to handle. Moreover, we establish results that identify the compromise effect under which a decision-maker has irrational choice behaviors. In the proposed aggregation function, the alternative utility is obtained as a weighted sum of multicriteria utilities. Our work provides continuation for the growing interest in developing MCDM with multicriteria interactions for various decision contexts. For instance, Labreuche and Grabisch (2006) developed a Choquet integral preference model for multiple criteria analysis, where the underlying scales were bipolar. Mayag and Bouyssou (2020) studied the interaction phenomena within a 2-additive Choquet integral model for the interpretation of decision making. Liao et al. (2020) produced a fuzzy Choquet integral operator to handle multicriteria interactions in fuzzy decision making. Singh and Kumar (2021) utilized a Choquet integral-based method to represent preferences in multicriteria group decision making problems.

In addition to the preference model, this paper also develops a preference learning approach that can be recast in terms of an optimization problem. The developed preference learning model is a variant of the general preference learning approaches, known as *learning to rank* in the field of machine learning (Hüllermeier et al. 2008; Bertsimas and O'Hair 2013; Aggarwal and Fallah Tehrani 2019). The basic idea of preference learning is to learn a preference model from decision-makers' preference information. Preference learning has been naturally used in MCDM (Tehrani et al. 2012). For an alternative a characterized by n criteria values, the utility value of alternative a can be modeled by an aggregation function $f(\cdot)$, such that

$$U(a) = f(a^{(1)},\ldots,a^{(n)}) \tag{1}$$

where $a^{(j)}$ is the evaluation value of a under the jth criterion. $f(\cdot)$ can be used to predict the preference of a decision-maker for any given pair of alternatives.

Preference learning models with robust outcomes has been discussed and developed by scholars (Bertsimas & O'Hair 2013; Corrente et al. 2013; Bouaziz et al. 2021). In particular, Corrente et al. (2013) developed the robust ordinal regression (ROR) for robust preference learning using a precise representation of preference relations. This framework differs from the conventional preference learning since it assumes there exists necessary and possible preference relations simultaneously. Hence, the ROR method may help offer a robust recommendation to the decision-maker for further interactivity with a learning model. The ROR model has been successfully integrated with MCDM methods (Corrente et al. 2016). Several studies (Greco et al. 2014; Angilella et al. 2016; Arcidiacono et al. 2020) used the ROR method to learn criteria interactions in MCDM problems. The focus of this paper is to develop an ROR preference learning model consistent with human behaviors and the criteria being not independent.

The contributions of this study are as follows:

(a) We incorporate the compromise effect into the preference model by inducing a concave aggregation operator.

(b) A generalized Choquet integral is proposed to aggregate the values of alternatives under multicriteria that interact with each other.

(c) The ROR preference learning model is developed to keep a balance between robustness and accuracy. The effectiveness of the proposed model is verified by a numerical study about the sustainable product evaluation.

There has also been considerable interest in applying multiple criteria analysis to evaluate sustainability among, for instance, biomass crop (Cobuloglu & Büyüktahtakın 2015), metropolitan cities (Carli et al. 2018), supply chain development (Mastrocinque et al. 2020), and building structures (Sánchez-Garrido et al. 2022). The resulting models were often referred to as "MCDM" which consider multiple conflicting criteria in decision making. Indeed, even a power plant allocation problem may include more than 30 criteria (Erol et al. 2014). Because sustainability evaluation methods were motivated by applications of MCDM involving multicriteria, they depended on decision-makers' preferences that determine how different criteria are valued. Many of these methods did not consider multicriteria interactions but rather assumed criteria are independent to each other. Further, in practice, decision-makers may have irrational choice behavior when facing a set of alternatives (Guo 2016; Howes et al. 2016; Aggarwal & Fallah Tehrani 2019). This paper addresses these limitations by applying a contextual Choquet integral-based preference learning model which can identify interactions between multicriteria and capture decision-makers' compromise effect.

The study is organized as follows: Sect. 2 reviews the state-of-the-art of aggregation operators as the background of the present study. Sect. 3 introduces the contextual Choquet integral (*CCI*) and present the *CCI* operator that adheres to human behaviors. An illustrative example is given to motivate the introduction of the new method. Sect.

4 presents a preference-learning model based on the optimization approach. Sect. 5 provides a numerical study with respect to the sustainable product evaluation. Sect. 6 describes the extensions of our learning framework to a further interactive process and the extension of the *CCI* operator to machine learning. The paper was closed in the last section.

2. Preliminaries

Applying the expected utility theory to decision analysis, we can characterize the preferences of a decision-maker through expected utility values. In the MCDM setting, a decision-maker may intuitively consider an aggregation process among the criteria utility values and determine her/his preferences among alternatives by the aggregated values. Specifically, this process can be typically characterized by an underlying aggregation function f : $\mathbb{R}^n \rightarrow \mathbb{R}$, such that for any pair of alternatives a and b, a is preferred to b in the eyes of the decision-maker if and only if $f(a) > f(b)$. The simplest preference model, assigning a real number to each alternative, has an additive structure defined as (Keeney & Raiffa 1976):

$$f(a) = \sum_{j=1}^{n} u_j(a) \tag{2}$$

where $u_j(a)$ is a marginal utility function with respect to the jth criterion and $j = 1, \ldots, n$. The preference model shown as Eq. (2) is intuitive to aggregate *n*-tuple input to derive the overall value $f(a)$ for alternative a.

Since the traditional additive structure failed to model multicriteria interactions, the Choquet integral (Chquet, 1954) has been taken as an alternative of the additive value function. Let $G = \{g_1, g_2, \ldots, g_n\}$ be a finite set of criteria. Consider an importance measure $\mu: 2^G \rightarrow [0,1]$ that returns a real value on the set *G*. $\mu(A)$ is interpreted as the weight of the set of criteria $A \subseteq G$. In the case of weighted arithmetic mean, the weights of coalitions can be represented in an additive form, *i.e.*, $\mu(\{g_1, g_2\}) = \mu(g_1) + \mu(g_2)$ where criterion g_1 and criterion g_2 do not interact with each other. However, in the real world, the assumption that criteria are completely independent is too strong to satisfy. A famous example illustrating the interactions among criteria has been described in Grabisch (1996): if a student is evaluated with respect to two subjects, such as mathematics and physics, then the importance of the coalition of these two subjects should be lower than the simple sum of the weights of them. The example was based on an intuitive assumption that students with good performance in mathematics would also perform good in physics, and vice versa. In this regard, the importance of two sets of criteria can be represented as $\mu(A \cup B) = \mu(A) + \mu(B) + I(A, B)$ where $I(\cdot,\cdot)$ refers to the interaction index.

Considering the limitation of additive models in expressing the real relations between decision criteria, the fuzzy measure (Sugeno, 1974) was introduced to represent the importance associated to a set of criteria, where

$$\mu(\varnothing)=0,\ \mu(G)=1 \text{ and } \mu(A)\geq\mu(B) \text{ if } B\subseteq A\subseteq G \tag{3}$$

Since it is used to represent the importance of a set of criteria, the fuzzy measure can capture the interactions of associated criteria. Yet, it is difficult to determine the interactive relations using the theoretical form in Eq. (3) directly. An alternative representation of the fuzzy measure is the *Möbius transform* (Rota 1964), which is a linear transform on the fuzzy measure:

$$\mu(B)=\sum_{A\subseteq B} m^{\mu}(A), \text{ for any } B\in 2^{G} \tag{4}$$

The invertible transform of the Möbius representation in Eq. (4) can be expressed as:

$$m(A)=\sum_{B\subseteq A}(-1)^{|A|-|B|}\mu(B) \tag{5}$$

where $|A|$ indicates the cardinality of the set A.

However, it is still difficult to manipulate the transformed form of fuzzy measure directly, because if there are multiple criteria involved, the fuzzy measure needs $2^{|G|}$ coefficients to be determined. It is reasonable to consider the interaction only between two criteria rather than among *n*-tuples of criteria in an MCDM process. In this regard, the 2-additive fuzzy measure was proposed to capture the situation where a decision-maker can give the information in terms of positive or negative interactions between pairs of criteria, shown as (Grabisch 1997b; Corrente et al. 2016):

$$\mu(B)=\sum_{g_j\in B} m(\{g_j\})+\sum_{\{g_j,g_k\}\subseteq B} m(\{g_j,g_k\}) \tag{6}$$

where g_j and g_k are two interactive criteria. In this way, the number of parameters reduces to $n+\binom{n}{2}$ in the 2-additive fuzzy measure and the conditions in Eq. (3) are rewritten as:

$$m(\varnothing)=0,\ \sum_{g_j\in G} m(\{g_j\})+\sum_{\{g_j,g_k\}\subseteq G} m(\{g_j,g_k\})=1$$

$$m(\{g_j\})\geq 0, \text{ and } m(g_j)+\sum_{g_k\in T} m(\{g_j,g_k\})\geq 0,\ \forall g_j\in G \text{ and } \forall T\subseteq G\setminus\{g_j\} \tag{7}$$

In MCDM problems, a decision-maker does not always treat the aggregation process as a weighted sum of utilities of different criteria but considers different combinations (coalitions) of criteria. That is to say, the decision-maker may consider interactions among criteria. Positive interactions increase the importance of a combination, while negative interactions reduce the importance of a combination. In this regard, the varying degrees of interactions can be represented by the fuzzy measure $\mu(\cdot)$, which induces two interaction indices as we show next.

Grabisch (1997b) introduced the concept of *Shapley value* to represent the criteria importance by considering the average contribution of a criterion to the whole set of criteria, which can be expressed by $\mu(\cdot)$. Given that μ is a fuzzy measure on G, the importance index of criterion g_j can be represented by the Shapley value (Shapley 1953):

$$\phi(\{g_j\}) = \sum_{A\subseteq G/\{g_j\}} \frac{(n-|A|-1)!|A|!}{n!}(\mu(A\cup\{g_j\})-\mu(A)), \text{ for } j=1,2,\ldots,n. \tag{8}$$

The Shapley value satisfies that $0 \le \phi(\{g_j\}) \le 1$ and $\sum_{j=1}^{n}\phi(\{g_j\})=1$. These properties are similar to the weights in conventional additive operators. Thus, $\phi(\{g_j\})$ has an intuitive interpretation as a measure of relative importance of criteria. For the 2-additive fuzzy measure, it can be written as:

$$\phi(\{g_j\}) = m(\{g_j\}) + \frac{1}{2}\sum_{g_k\in G/\{g_j\}} m(\{g_j,g_k\}) \tag{9}$$

On the other hand, the *interaction index* $I(\{g_j,g_k\})$, taking into account the interaction between two criteria g_j and g_k, is proposed by Murofushi and Soneda (1993) and defined as follows:

$$I(\{g_j,g_k\}) = \sum_{A\subseteq G\setminus\{g_j,g_k\}} \frac{(n-|A|-2)!|A|!}{(n-1)!}(\mu(A\cup\{g_j,g_k\}-\mu(A\cup\{g_j\})-\mu(A\cup\{g_k\})+\mu(A)) \tag{10}$$

where $I(\{g_j,g_k\}) \in$ [-1,1], indicating the positive (resp. negative) interaction if $I(\{g_j,g_k\})>0$ (resp. $I(\{g_j,g_k\})<0$). Furthermore, the interaction index can be rewritten with the aid of the Möbius transform:

$$I(\{g_j,g_k\}) = \sum_{T\subseteq G\setminus\{g_j,g_k\},} \frac{1}{|T|+1} m(\{g_j,g_k\}\cup T) \tag{11}$$

Choquet integral is a popular operator to capture the interactions among criteria. Given a fuzzy measure μ, the Choquet integral is a mapping C_μ: $[0,1]^n \to [0,1]$ that integrates n-tuple values into a comprehensive value. The Choquet integral of $x=(x_1,\ldots,x_n)\in\mathbb{R}^n$ is defined as:

$$C_\mu(x) = \sum_{j=1}^{n} x_{\sigma(j)}(\mu(A_j)-\mu(A_{j+1})) \tag{12}$$

where $\sigma(\cdot)$ is a permutation of the indices of criteria such that $x_{\sigma(1)} \le \ldots \le x_{\sigma(n)}$, $A_j=\{x_{\sigma(j)},\ldots,x_{\sigma(n)}\}$ and $A_{n+1}=\varnothing$. A_j is the subset of $n-j+1$ largest components in x.

In MCDM, the utility of alternative a against criterion g_j is identified as $f_j(a)$. The aggregated utility values using the fuzzy measure μ can be represented by an aggregation function $C_\mu(f)$, shown as:

$$C_{\mu}\left(f_1(a),\ldots,f_n(a)\right)=\sum_{j=1}^{n} f_{\sigma(j)}(a)(\mu(A_j)-\mu(A_{j+1})) \tag{13}$$

where $\sigma(\cdot)$ indicates a permutation of the criteria such that $0\le f_{\sigma(1)}(a)\le\ldots\le f_{\sigma(n)}(a)$.

For independent criteria, the Choquet integral reduces to the weighted mean operator, represented as:

$$C_{\mu}\left(f_1(a),\ldots,f_n(a)\right)=\sum_{j=1}^{n} f_j(a)\cdot\mu(\{g_j\})=\sum_{j=1}^{n} f_j(a)\cdot w_j \tag{14}$$

where w_j is the weight of the criterion g_j. In this case, $w_j=\mu(\{g_j\})$ and $\mu(\{g_j\})$ indicates the importance of g_j. An alternative form of Eq. (14) can be deduced from the weighted sum with the fuzzy measure as follows:

$$\begin{aligned}C_{\mu}\left(f_1(a),\ldots,f_n(a)\right)&=\sum_{j=1}^{n} f_j(a)\cdot\mu(\{g_j\})\\&=\sum_{j=1}^{n}(f_{\sigma(j)}(a)-f_{\sigma(j-1)}(a))\cdot(\mu(\{g_{\sigma(j)}\})+\ldots+\mu(\{g_{\sigma(n)}\}))\\&=\sum_{j=1}^{n}(f_{\sigma(j)}(a)-f_{\sigma(j-1)}(a))\cdot\mu(A_j)\end{aligned} \tag{15}$$

where $A_j=\{f_{\sigma(j)}(a),\ldots,f_{\sigma(n)}(a)\}$ and $f_{\sigma(0)}=0$. Using the Möbius representation of fuzzy measure, we present the Choquet integral in Eq. (15) as follows:

$$C_{\mu}\left(f_1(a),\ldots,f_n(a)\right)=\sum_{T\subseteq G} m(T)\min_{g_j\in T} f_j(a) \tag{16}$$

We will use this form in the optimization problem. To illustrate the computation of the Choquet integral, we give a simple example that a house buyer evaluates the house a considering its performances on two criteria, prize and size, in [0, 1] scale. The evaluations of the house are expressed as follows: $f_{price}(a)=0.6$, $f_{size}(a)=0.8$. To represent the criteria importance and interaction, the buyer sets $\mu(\{price\})=0.4$, $\mu(\{size\})=0.3$, $\mu(\{price,\ size\})=1$. Using Eq. (15), the aggregated value of a by the Choquet integral is computed as:

$$C_{\mu}\left(a\right)=f_{price}(a)\cdot\mu(\{price,\ size\})+(f_{size}(a)-f_{price}(a))\cdot\mu(\{size\})=0.66\,. \tag{17}$$

The Möbius representation m in terms of the fuzzy measure μ can be given as $m(\{price\})=0.4$, $m(\{size\})=0.3$, $m(\{price,size\})=0.3$. $m(\{price,size\})>0$ denotes the positive interaction between the criteria price and size. The preceding computation can be rewritten by Möbius transformations as:

$$C_{\mu}\left(a\right)=f_{price}(a)\cdot m(\{price\})+f_{size}(a)\cdot m(\{size\})+\min(f_{price}(a),\ f_{size}(a))m(\{price,size\})=0.66\,. \tag{18}$$

3. A contextual Choquet integral preference model

According to behavioral economics, decision-makers do not always adhere to the law of value maximization, rather often choose a nonoptimal alternative. The conventional preference models incorporating the Choquet integral, are based on the value maximization assumption, which cannot model the preference reversal phenomena in the real world. In other words, the Choquet integral can consider the interactions of criteria but ignores the behavioral characteristics of individuals. In this regard, inspired by the compromise effect reported in the literature (see Simonson and Tversky 1992; Guo 2016; Howes et al. 2016), this section proposes a contextual Choquet integral-based preference model which takes into account not only the criteria interactions but also the behavioral characteristics in human decision making processes.

The contextual Choquet integral is a generalization of the classical Choquet integral and is capable of modeling the behavioral characteristics of a decision-maker regarding criteria interactions. Our approach assumes that the aggregation function has a parametric structure to represent the decision-maker's behavioral characteristics through the degree of compromise effect which can be characterized by a concave function that maps criteria values to psychological responses (Kivetz et al. 2004). In other words, the aggregation process considers the effect of irrelevant alternatives in pairwise comparisons. Depending on different parameter values, we can obtain different aggregated scores in two comparisons for the same pair of alternatives, compatible with the compromise effect (Tversky and Simonson 1993). We propose an aggregation operator called the "*contextual concavity*" by combining the notions of context dependence and concavity mathematically.

Let $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ be a set of alternatives and $S \subseteq \mathcal{A}$ be a reference set included in the whole set. $a \succ b$, for any $a, b \in S$, means a preference that the decision-maker prefers a to b. Formally, the *contextual Choquet integral* is defined as a mapping *CCI*: $[0,1]^n \to [0,1]$, given by:

$$CCI(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} (\mu(A_j) - \mu(A_{j+1})) \cdot (x_{\sigma(j)} - x_{\sigma(j)}^{\min,S})^{\lambda} \tag{19}$$

where $x_{\sigma(j)}^{\min,S}$ is the lowest value on criterion $g_{\sigma(j)}$ for a reference set S and λ is the concavity parameter with $\lambda > 0$.

The contextual concavity leads to a concave aggregation function on the consequences, such as the power function in Eq. (19) where $0 < \lambda < 1$. Eq. (19) uses fuzzy measure μ and parameter λ to capture the mentioned two effects, where μ represents the degree of interaction and λ indicates the degree of compromise effect of the decision-maker. λ reveals the sensitivity degree of the decision-maker on the extreme values of criteria. For an extremeness seeking decision-maker (Simonson and Tversky 1992), the extreme alternative with maximum or minimum criteria values is more desirable than intermediate ones. Conversely, an extremeness aversion decision-maker displays a preference for intermediate alternatives. That is, the decision-maker makes trade-off among criteria and choose an alternative with the best overall performance on all criteria. A bad performance on a criterion would lead to a great reduction on the overall score and gives rise to the preference reversal phenomenon.

An equivalent representation of *CCI* in terms of the Möbius transform is expressed as:

$$CCI(x_1, x_2, \ldots, x_n) = \sum_{T \subseteq G} m(T) \cdot \min_{j \in T} (x_j - x_j^{\min,S})^{\lambda} \ . \tag{20}$$

If we represent the criteria values by utility functions, then, Eq. (20) can be rewritten as:

$$CCI(f_1(a), f_2(a), \ldots, f_n(a)) = \sum_{T \subseteq G} m(T) \cdot \min_{j \in T} (f_j(a) - f_j^{\min,S}(a))^{\lambda} \tag{21}$$

where $f_j^{\min,S}(a)$ indicates the minimum utility value of alternative a under criterion g_j for the reference set S.

From the preference-learning point of view, learning the parameters in *CCI* from available preferences to estimate the value function is the main concern. Next, we provide several intuitive observations with respect to the concavity parameter λ prior to the introduction of our learning model. In many cases, λ is expected to be smaller than one as the compromise effect refers to a concave function. In other words, *CCI* can explain the compromise effect only when $\lambda < 1$. In this case, the lower parameter value implies that the decision-maker is more sensitive to the extremeness aversion. However, the impact would reduce in the situation where the decision-maker has clear preferences and positive responses to uncertainty. When $\lambda = 1$, $CCI = \sum_{j=1}^{n} (\mu(A_j) - \mu(A_{j+1})) \cdot (x_{\sigma(j)} - x_{\sigma(j)}^{\min,S})$ reduces to the conventional Choquet integral with normalized criteria values. If $\lambda > 1$, Eq. (19) would constitute an extremeness seeking preference structure, which implies that the decision-maker concentrates on the alternatives with extremely good performances on some criteria.

In the real world, the behavioral characteristics largely impact decision-maker's preferences and in particular may cause the preference reversal. Interaction among criteria is another important consideration in MCDM problems. In this regard, the CCI operator models these two effects at the same time. To explain the effectiveness of the *CCI*, this section gives an example concerning the behavior of a house-buyer and then compare the CCI with the conventional Choquet integral.

Example 1. Suppose that a house-buyer wants to choose a house from three alternatives which are evaluated on *price*, *size* and *comfort*, as shown in Table 1. Without loss of generality, we define all criteria on a common gain scale [0, 1]. The output is an overall score between zero and one, corresponding to the degree of an individual's preference to a house. The buyer considers that the three criteria have the same importance; the criteria *size* and *comfort* have negative interaction; the remaining pairs of criteria have positive interaction. For the sake of simplicity, we assume that the Möbius values, which will be determined by the preference-learning technique described in the next section, are shown in Table 2. In this example, we consider the *CCI* with parameter $\lambda = 0.473$ ($\lambda < 1$). This is because a previous behavioral science experiment had shown that such a value can distinctly capture the contextual concavity in decision-makers' preference structure (Kivetz et al. 2004). We shall use the same data in the classical Choquet integral model that only captures criteria interactions but not the compromise effect.

Table 1. Evaluations of the houses on three criteria

House	*Price* (*Pr*)	*Size* (*Si*)	*Comfort* (*Co*)
H_1	0.6	0.9	0.8
H_2	0.7	0.8	0.8
H_3	0.8	0.7	0.8

Table 2. Möbius representation of the fuzzy measure

$m(\{Pr\})$	$m(\{Si\})$	$m(\{Co\})$	$m(\{Pr, Si\})$	$m(\{Pr, Co\})$	$m(\{Si, Co\})$
0.3	0.3	0.3	0.3	0.1	-0.3

By Eqs. (16) and (21), we can compute the aggregated scores of the three alternatives. Table 3 lists the aggregated scores with respect to the weighted arithmetic mean, the classical Choquet integral and the *CCI*. Taking H_1 as an example, using the *CCI*, the score of H_1 is obtained as follows:

$$\begin{aligned} CCI(H_1) &= 0.3 \times \min(\{0^{0.473}\}) + 0.3 \times \min(\{0.2^{0.473}\}) + 0.3 \times \min(\{0^{0.473}\}) + 0.3 \times \min(\{0^{0.473}, 0.2^{0.473}\}) \\ &\quad + 0.1 \times \min(\{0^{0.473}, 0^{0.473}\}) - 0.3 \times \min(\{0.2^{0.473}, 0^{0.473}\}) = 0.140 \end{aligned}$$

Table 3. Aggregated scores based on the weighted arithmetic mean, Choquet integral and *CCI*

House	Weighted arithmetic mean	Choquet integral	*CCI*
H_1	0.77	0.69	0.140
H_2	0.77	0.73	0.303
H_3	0.77	0.77	0.140

From Table 3, it is clear that the Choquet integral imply $H_3 \succ H_2 \succ H_1$. Conversely, the *CCI* leads to the reversal preferences $H_1 \prec H_2 \succ H_3$.

Figure 1 illustrates how the *CCI* operator captures the compromise effect but the classical Choquet integral might fail to. The *CCI* is better for capturing decision-makers' behavioral characteristics concerning the preferences of intermediate alternatives. It can be attributed to the flexibility of this model in rescaling the original utility space. That is, the *CCI* can change the aggregation process by rescaling the criteria values with an adjustable parameter. For instance, the decision-maker's difference between the score obtained from an intermediate alternative and that derived from extreme alternatives can be captured by the *CCI*. Panel A indicates the utility values under the criterion

Pr, and panel B indicates the utility values under the criterion *Si*. For simplicity of exposition, we assume that the decision-maker has linear functions on criterion levels. Therefore, in panels A and B of Fig. 1, there are two straight lines (green ones), which represent the linear utilities used by the classical Choquet integral. However, The concave lines (orange ones) used by *CCI* operator represent the utility values of an extremeness-aversion decision-maker associated with a concavity parameter $\lambda < 1$. In panel C of Fig. 1, using classical Choquet integral and *CCI*, we aggregate the utility values across the two criteria to form the graphs of the overall scores of the alternatives. Fig. 1 reveals an apparent preference reversal ($H_2 \succ H_3$) using the *CCI* operator. As we can see, the *CCI* is convenient to represent the compromise effect in the decision process, thus resulting in a flexible model compared with the classical Choquet integral.

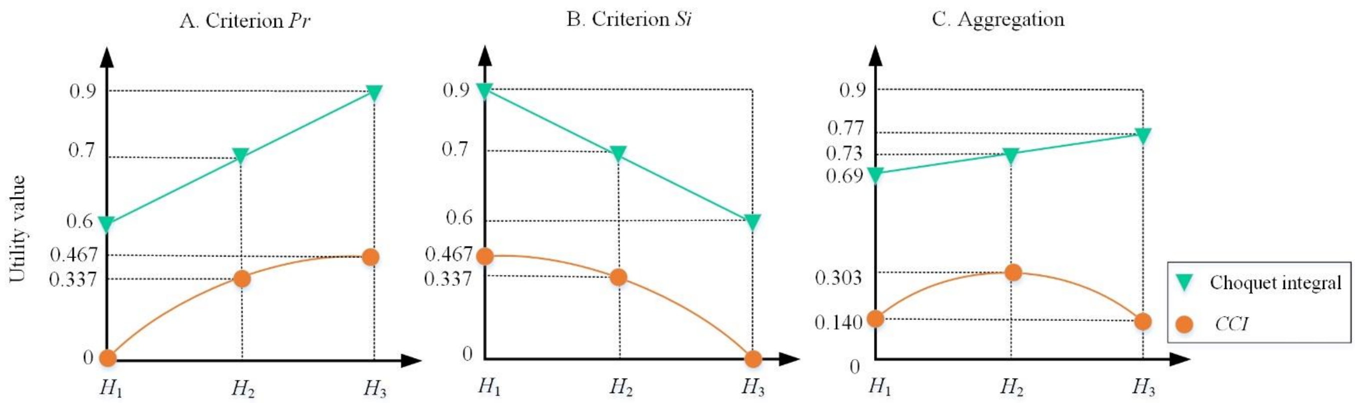


Fig. 1. The aggregation process modeled by the *CCI* operator and the classical Choquet integral operator

Notes: x axis denotes the three alternatives (H_1, H_2, H_3), and y axis denotes the utility values or aggregated scores of the alternatives.

More generally, the *CCI* operator is better to represent the decision-makers' preferences with respect to the intermediate alternatives against the extreme alternatives than the classical Choquet integral. Considering the invalidity of using the classical Choquet integral in the previous example, the house-buyer might wonder whether it is possible to capture her/his preferences through changing the interaction indices among the three criteria. Unfortunately, the classical Choquet integral cannot represent the preferences of the house-buy with respect to the intermediate alternatives. Indeed, aggregating the utility values by the classical Choquet integral should imply that

$$CI(H_2) = 0.7m(\{Pr\}) + 0.8m(\{Si\}) + 0.8m(\{Co\}) + 0.7m(\{Pr, Si\}) + 0.7m(\{Pr, Co\}) + 0.8m(\{Si, Co\})$$

$$CI(H_3) = 0.8m(\{Pr\}) + 0.7m(\{Si\}) + 0.8m(\{Co\}) + 0.7m(\{Pr, Si\}) + 0.8m(\{Pr, Co\}) + 0.7m(\{Si, Co\})$$

By the difference of the two aggregated scores we have

$$CI(H_2) - CI(H_3) = -0.1m(\{Pr, Co\}) + 0.1m(\{Si, Co\})$$

and clearly, since $m(\{Pr,Co\})>0$ and $m(\{Si,Co\})<0$, $H_2 \prec H_3$ ($CI(H_2)<CI(H_3)$) for all $m(A)$, $A\in 2^G$. In that case, the preferences on the intermediate alternatives (or compromise effect) cannot be represented by the classical Choquet integral. Geometrically speaking, the *CCI* adds the additional concavity on the context-independent Choquet integral (see Eq. 13). That is, regardless the initial shape of a classical Choquet integral operator, the *CCI* adds another layer of concavity based on the context. As a result, in the process of aggregation, the overall score of an alternative would be changed a lot and eventually cause the preference reversal (*e.g.*, $H_1 \prec H_2 \succ H_3$), which is consistent with the hypothesis of the extremeness aversion[1] .

4. A learning model with robust ordinal regression

From Section 3.1, we can find that the magnitude of the context dependence of a decision-maker is represented by the concavity parameter λ ($\lambda<1$). In such a case, λ reshapes the utility function in Eq. (13) and eventually gives rise to the concavity on the aggregation function. Depending on different values of λ, the same set of input data can be integrated into a range of scores. This motivates us to learn the aggregation function *CCI* that is characterized by parameters to represent a decision-maker's specific sensitive degree of extremeness aversion. In this section, we try to achieve this goal of parameter learning through a recently emerging technique, the ROR.

4.1. A robust ordinal regression model

The ROR is a special preference learning-based technique to offer robust results in terms of preference relations. The main idea of this approach is to use all compatible value functions rather than to arbitrarily select a single value function to represent the decision-maker's preferences, such that all the known preference information can be used in the final decision. Inspired by this idea, we use the parametric value function, *i.e.*, the *CCI*, for preference learning.

Consider a reference set $S \subseteq \mathcal{A}$ where each alternative $a\in S$ is defined on n criteria, denoted by $G=\{1,\ldots,n\}$. We define a set of preference tuples as $\mathcal{R}=\{(a_i,b_i)\,|\,a_i \succ b_i, 1\le i\le M\}$, where M is the number of preference pairs generated from *S*, taken as the inputs of our model. Furthermore, considering the criteria interactions marked by the interaction index given in Eq. (11), one can easily incorporate the preference information of criteria (*i.e.*, the interaction and the importance of criteria) into the learning problem, fed also as inputs of our learning model. The learning system is to infer the *CCI* that can reproduce the decision-maker's preferences. Such a set of value functions

[1] Tversky and Simonson (1993) claimed that decision makers often choose intermediate options rather than extreme ones. They interpret this phenomenon as "extremeness aversion" which leads to two effects: compromise and polarization.

is called *compatible value functions*. From the optimization point of view, we aim to determine all parameters μ of the compatible value functions such that the resultant preference relations are the most representative recommendations for input data.

To do so, we choose the *CCI* as an aggregation operator to approximate decision-makers' preferences. In the MCDM context, we consider that the *CCI* is defined on a set of criteria $G = \{1, \dots, n\}$ and characterized by the fuzzy measure $\mu(\cdot)$. In this sense, the Möbius transform given as Eq. (4) would be used in the model because of its convenience of computation. To learn the values of $m(\cdot)$, we consider a set of constraints E^S. Formally, define the constraints E^S as

$$\left.\begin{array}{l}
CCI_{\mu,\lambda}(a_i) > CCI_{\mu,\lambda}(b_i), \text{if } a_i \succ b_i \\
CCI_{\mu,\lambda}(a_i) = CCI_{\mu,\lambda}(b_i), \text{if } a_i \sim b_i \\
\lambda > 0 \qquad\qquad\qquad \lambda \neq 1 \\
\phi(j) > \phi(k), \text{if } j \succ^* k \\
\phi(j) = \phi(k), \text{if } j \sim^* k \\
I(\{j,k\}) > 0, \text{if } j \text{ interacts with } k \text{ positively} \\
I(\{j,k\}) < 0, \text{if } j \text{ interacts with } k \text{ negatively} \\
m^{\mu}(\varnothing) = 0 \\
\sum_{j \in G} m^{\mu}(\{j\}) + \sum_{\{j,k\} \subseteq G} m^{\mu}(\{j,k\}) = 1 \\
m^{\mu}(\{j\}) \geq 0, \text{ and } m^{\mu}(\{j\}) + \sum_{k \in T} m^{\mu}(\{j,k\}) \geq 0, \forall j \in G \text{ and } \forall T \subseteq G \setminus \{j\}
\end{array}\right\} E^S \tag{22}$$

where $\succ^*$ is defined on $G \times G$, denoting the preferences on criteria importance. That is, for criteria $j, k \in G$, if $j \succ^* k$, then it could indicate that criterion j is more important than criterion k. Note that the value of λ depends on the compromise effect that in decision-maker's preference behavior, which is required to be specified beforehand. Hence, for constraints (22), λ is a constant that is essentially different from the tuning hyperparameter used in machine learning.

To verify whether there exists a compatible value function, we introduce a slack variable for a transformation of the strict inequalities in E^S. This leads to the following optimization problem:

$$\varepsilon^* = \max \varepsilon$$

$$
\left.\begin{array}{ll}
CCI_{\mu,\lambda}(a_i) \geq CCI_{\mu,\lambda}(b_i) + \varepsilon, \text{ if } a_i \succ b_i & \forall (a_i, b_i) \in \mathcal{R} \\
CCI_{\mu,\lambda}(a_i) = CCI_{\mu,\lambda}(b_i), \text{ if } a_i \sim b_i & \forall i \in \{1,\ldots,M\} \\
\lambda > 0 \qquad\qquad \lambda \neq 1 & \\
\phi(j) \geq \phi(k) + \varepsilon, \text{ if } j \succ^* k & \\
\phi(j) = \phi(k), \text{ if } j \sim^* k & \\
I(\{j,k\}) \geq \varepsilon, \text{ if } j \text{ interacts with } k \text{ positively} & \\
I(\{j,k\}) \leq -\varepsilon, \text{ if } j \text{ interacts with } k \text{ negatively} & \\
m^\mu(\varnothing) = 0 & \\
\sum_{j \in G} m^\mu(\{j\}) + \sum_{\{j,k\} \subseteq G} m^\mu(\{j,k\}) = 1 & \\
m^\mu(\{j\}) \geq 0, \text{ and } m^\mu(j) + \sum_{k \in T} m^\mu(\{j,k\}) \geq 0, \forall j \in G \text{ and } \forall T \subseteq G \setminus \{j\} &
\end{array}\right\} E^{S'} \tag{23}
$$

where ε is an arbitrarily small positive value used to transform the strict inequality into weak inequality constraints. $E^{S'}$ is to convert the preference statements of a decision-maker into an evaluation space. We have a set of compatible value functions $\mathcal{CCI}_{\mu,\lambda}$ if there exists a nonempty polytope defined by the set of constraints $E^{S'}$; that is, if $\varepsilon^* > 0$, there exists at least one capacity compatible with the decision-maker's preferences. For an empty polytope, one can conveniently resolve inconsistencies among the constraints through accessible methods among which the most popular one is the 0-1 mixed integer programming-based method (Greco et al. 2010).

Each compatible value function may return different total rankings when there exists more than one value function. In this regard, we find the emerging ROR technique potentially useful for this arbitrariness (see Greco et al. 2008), which considers all compatible value functions instead of using a single value function. This helps to apply all the preference information of a decision-maker to reach a final recommendation through the *necessary* and *possible* preference relations.

4.2. A preference learning model

A robust preference learning is to distinguish necessary and possible preference relations by a set of compatible value functions instead of using a single value function with optimal parameters. Corrente et al. (2013) showed that for a set of plausible values of parameters of the value function, the ROR technique can lead to robust preference relations for a set of alternatives. In particular, the robustness emphasizes a necessary preference such that the necessary preference of any pair of alternatives is stable across the compatible value functions, in spite of slight variations among the value functions. On the other hand, it diminishes the possible preferences when new preference information is added to the reference set. From a preference-learning viewpoint, this significantly facilitates the learning of robust rankings.

In the context of MCDM, we consider a finite number of pairwise preferences in the form of $(a^*,b^*) \in \mathcal{A} \times \mathcal{A}$. With the compatible value functions obtained from the preceding optimization problem, we here add a new constraint of each preference tuple (a^*,b^*) such that $a^* \succsim b^*$ if $CCI_{\mu,\lambda}(a^*) \geq CCI_{\mu,\lambda}(b^*)$, where $CCI_{\mu,\lambda} \in \mathcal{CCI}_{\mu,\lambda}$. Based on the new constraints, we define two different binary relations that are able to be inferred from the learned compatible value functions, shown as:

- The necessary preference relation $\succsim^N$, such that $a^* \succsim^N b^*$ iff for all compatible value functions, $CCI_{\mu,\lambda}(a^*) \geq CCI_{\mu,\lambda}(b^*)$.
- The possible preference relation $\succsim^P$, such that $a^* \succsim^P b^*$ iff for at least one compatible value function, $CCI_{\mu,\lambda}(a^*) \geq CCI_{\mu,\lambda}(b^*)$.

The main idea of our model is to distinguish two preference relations so as to derive a robust ranking of the given alternatives. To this end, we check the feasibility of the constraints by an optimization process which minimizes the misranking error. The margin of the misranking error is determined by maximizing the slack variable ε. Specifically, we maximize the margin between the aggregated scores of alternatives a^* and b^* for a preference tuple $a^* \succsim b^*$. In this regard, we consider an additional pairwise comparison in Model (23) to verify the preference relation of (a^*,b^*) through all compatible value functions $CCI_{\mu,\lambda} \in \mathcal{CCI}^S_{\mu,\lambda}$.

Formally, check the necessary relation by solving the following optimization problem:

$$\begin{aligned} &\varepsilon_N^* = \max \varepsilon \\ &s.t.: \left.\begin{matrix} CCI_{\mu,\lambda}(b^*) \geq CCI_{\mu,\lambda}(a^*) + \varepsilon \\ E^{S'} \end{matrix}\right\} E_N^{S'} \end{aligned} \tag{24}$$

Similarly, check the possible relation by solving the following optimization problem:

$$\begin{aligned} &\varepsilon_P^* = \max \varepsilon \\ &s.t.: \left.\begin{matrix} CCI_{\mu,\lambda}(a^*) \geq CCI_{\mu,\lambda}(b^*) \\ E^{S'} \end{matrix}\right\} E_P^{S'} \end{aligned} \tag{25}$$

where ε is a margin that is expected to be maximized. If $\varepsilon_N^* \leq 0$ or $E_N^{S'}$ is infeasible, we have a necessary preference relation that $a^* \succsim^N b^*$. If $\varepsilon_P^* > 0$ or $E_P^{S'}$ is feasible, we have a possible preference relation that $a^* \succsim^P b^*$. $\varepsilon_N^* \leq 0$ implies that the preference tuple of (a^*,b^*) can be characterized by all compatible value functions and $\varepsilon_P^* > 0$ indicates that the preference tuple of (a^*,b^*) can be identified by at least one value function. Compared with the necessary relation, the possible relation cannot reproduce the decision-maker's preferences with all value functions in $\mathbb{CCI}_{\mu,\lambda}$. The above optimization problems involve a set of nonlinear constraints generated by the preference tuples of alternatives.

We note that $\varepsilon_N^* \le 0$ indicates that the new additional inequality in Model (24) cannot be satisfied. In other words, for a preference tuple $b^* \succsim a^*$, the necessary preference relations would be characterized by an empty polytope of the constraints for all compatible value functions. Consequently, the necessary outputs are robust to the partial preference information of a decision-maker, thus compatible with all instances. Conversely, the possible relation assumes that the new additional inequality in Model (25) can be achieved for at least one compatible value function. This model ensures a robustness analysis instead of finding an optimal solution by a single value function. As a result, the preference-learning problem is accomplished by two independent optimization problems.

In summary, we conduct the preference learning process based on the *CCI* in the following steps:

Step 1. Select a set of reference alternatives S from the whole set $\mathcal{A}$. S is chosen as a subset that provides a good balance between the ability of prediction and cognitive costs in the context-dependent choice.

Step 2. From S, the decision-maker generates M preference tuples in the form (a_i, b_i), such that $a_i \succsim b_i$. The decision-maker is also required to provide information of the multicriteria interactions.

Step 3. An analyst is involved in the decision process to observe the compromise effect and specify the concavity parameter λ.

Step 4. The preference-learning model presented in Model (23) is solved on S to learn a set of compatible value functions.

Step 5. Analyst checks the preference inconsistency and decides whether to adjust the preference information or not. If it needs to be revised, go to the next step; otherwise, we skip to the last step.

Step 6. The inconsistent pairwise comparisons (constraints $E^{S'}$) are removed iteratively by following the procedure recommended by Greco et al. (2010) until there exists no inconsistent constraint.

Step 7. Based on the compatible values functions, a robust decision recommendation is obtained by solving the optimization problems in Models (24) and (25).

5. Numerical study concerning sustainable product evaluation

In this section, we consider a sustainable product evaluation problem of the apparel industry to illustrate that our model can accommodate the compromise and interaction effects in the MCDM process. Since we intend to learn the human decision behavior, a decision-maker would be invited to provide the preferences on the reference set $S \subseteq \mathcal{A}$. The reference set can be chosen from the whole set randomly or given by an analyst.

5.1. Data collection

In this section, we apply the proposed preference learning model to a problem of sustainable product evaluation based on the data set originally presented by Gloria et al. (2013). The data were collected from the Levi's, one of the largest apparel corporations in the world, in terms of the life cycle environmental impact of a garment product. We focus on six impact criteria that are scaled to real numbers from 0 to 1. Although two criteria in the original matrices were not included, our present analysis may be effective because the results are not related to these criteria as shown in the original research. In this study, we consider 26 fabrics that were originally used by the Levi's E-valuate™ method to perform a life cycle assessment (LCA). We normalize the original data in Table 4 (for details, see Gloria et al. 2013). The values of the six life cycle impact criteria were collected from the garment mills on the material efficiency, water use, carbon emissions, land transformation and others. The six input criteria are described as follows:

- **Energy Use (EU),** refers to the cumulative energy demand (MJ) in the manufacturing activities of producing the fabric.
- **Water Use (WU),** refers to the freshwater usage (m^3) minus the returned water with the same quality as the input freshwater.
- **Global Warming Potential (GWP),** is measured by the greenhouse gas emissions (kg CO_2-e) that is responsible for the climate change.
- **Eutrophication (ET),** is measured by the phosphate emissions that lead to eutrophication (kg PO_4-e) of the body of water.
- **Land Occupation (LO),** refers to the land use (m^2-year) of the manufacturing system, such as fiber production, yarn spinning, weaving, dyeing, cutting, sewing, finishing, and the transportation from raw material extraction through the factory gate.
- **Abiotic Depletion (AD),** is measured by the nonrenewable resources (kg Sb-e) such as metals, conflict minerals, and fossil fuel.

Table 4. The LCA values of 26 denim fabrics

Fabric	EU	WU	GWP	ET	LO	AD
a_1	0.5	0.5	0.5	0.2	0.3	0.5
a_2	0.5	0.5	0.4	0.2	0.3	0.5
a_3	0.9	0.9	0.9	0.8	0.9	0.9

a_4	0.8	0.7	0.8	0.8	0.5	0.8
a_5	0.9	0.9	0.9	0.9	0.8	0.9
a_6	0.9	0.9	0.9	0.7	1.0	0.9
a_7	0.9	0.9	0.9	0.8	0.9	0.9
a_8	0.9	0.9	0.9	0.9	0.8	0.9
a_9	0.8	0.9	0.8	0.8	0.8	0.8
a_{10}	0.7	0.9	0.6	0.5	0.8	0.7
a_{11}	0.7	0.9	0.6	0.4	0.9	0.7
a_{12}	0.7	0.9	0.6	0.6	0.7	0.7
a_{13}	0.4	0.0	0.5	0.6	0.7	0.4
a_{14}	0.1	0.9	0.1	0.0	0.8	0.0
a_{15}	0.0	0.9	0.1	0.0	0.8	0.0
a_{16}	0.0	0.9	0.0	0.0	0.8	0.0
a_{17}	0.4	0.0	0.4	0.3	0.7	0.4
a_{18}	0.4	0.0	0.4	0.4	0.6	0.4
a_{19}	0.4	0.0	0.4	0.2	0.8	0.4
a_{20}	0.8	0.8	0.8	0.6	0.8	0.8
a_{21}	0.7	0.5	0.6	0.5	0.0	0.6
a_{22}	0.6	0.3	0.7	0.8	0.9	0.6
a_{23}	0.7	0.5	0.7	0.8	0.9	0.7
a_{24}	0.7	0.5	0.7	0.6	0.3	0.7
a_{25}	0.7	0.5	0.7	0.6	0.4	0.7
a_{26}	0.8	0.9	0.8	0.8	0.8	0.8

Notes: Gloria et al. (2013) accessed the environmental impact of producing a square yard of denim fabric, and the evaluations refer to the material inputs, land use, resource extractions, and other emissions. Subsequently, they rescaled the assessments to the integer values from 0 to 10. Without loss of generality, we normalize the real data to the scale from 0 to 1.

Suppose that a decision-maker of this corporation is invited to provide the preference information on the reference set $S \subseteq \mathcal{A}$. S is defined as a subset of alternatives, for which the decision-maker can express her/his preferences on the subset. Note that the preference information is often incomplete because it is very hard for a decision-maker to identify all preferences of alternatives, even deciding the binary comparisons between alternatives is difficult in some situations. The decision-maker is also expected to provide preference information about the interactions between each pair of criteria as much as possible.

The goal of preference learning is to translate the objective and subjective input data into the outcomes that the decision-maker can use directly. In this study, the preference learning can be accomplished by the decision-support model suggested in Section 4.2. The sustainability input data of this learning model are twofold: the objective utility values measured by the environmental impacts in the life cycle of a product and the subjective judgments with respect to individual's preferences on the products and criteria. In this regard, a decision-maker of the Levi's corporation was provided a reference set $S = \{a_3, a_5, a_6, a_7, a_8\}$[2] and subsequently asked to determine the preference information, shown as follows:

- The preference information of holistic binary comparisons of the five denim fabrics:
  - a_3 is better than a_5;
  - a_3 is better than a_6;
  - a_7 is better than a_6;
  - a_7 is better than a_8.
- The interaction information among couples of criteria:
  - criterion EU interacts with criteria GWP, ET and AD negatively;
  - criterion WU interacts with criteria LO and AD positively.

Besides uncertain preferences, our proposed model helps us understand the unique behavior of the decision-maker and deduce robust results, which can be illustrated in the next subsection.

5.2. Applying the proposed learning model

We perform the preference-learning model given in Section 4.3. Steps 1 and 2 have been accomplished through a preference elicitation process that requires the decision-maker to provide preference information as the input data. Therefore, we conduct the following steps in this section.

To check whether there exists at least one compatible value function based on the decision-maker's preferences, one can solve the optimization problem as follows:

$$\varepsilon^* = \max \varepsilon$$

---

[2] Considering the cognitive burden of the decision-maker, we select a reference set that consists of five alternatives having the same criteria values on four criteria and different values on the remaining two criteria. Particularly, to learn the underlying compromise effect, the decision-maker would be required to provide preferences on those nearly indifferent alternatives.

$$\left.\begin{array}{l} CCI_{\mu,\lambda}(a_3) \geq CCI_{\mu,\lambda}(b_5)+\varepsilon, \quad CCI_{\mu,\lambda}(a_7) \geq CCI_{\mu,\lambda}(b_6)+\varepsilon \\ CCI_{\mu,\lambda}(a_3) \geq CCI_{\mu,\lambda}(b_6)+\varepsilon, \quad CCI_{\mu,\lambda}(a_7) \geq CCI_{\mu,\lambda}(b_8)+\varepsilon \\ \lambda > 0 \\ I(\{\mathrm{WU},\mathrm{LO}\}) \geq \varepsilon, \quad I(\{\mathrm{WU},\mathrm{AD}\}) \geq \varepsilon \\ I(\{\mathrm{EU},\mathrm{ET}\}) \leq -\varepsilon, \quad I(\{\mathrm{EU},\mathrm{AD}\}) \leq -\varepsilon \\ I(\{\mathrm{EU},\mathrm{GWP}\}) \leq -\varepsilon \\ m^{\mu}(\varnothing)=0 \\ \sum_{j\in G} m^{\mu}(\{j\}) + \sum_{\{j,k\}\subseteq G} m^{\mu}(\{j,k\}) = 1 \\ m^{\mu}(\{j\}) \geq 0, \text{ and } m^{\mu}(j) + \sum_{k\in T} m^{\mu}(\{j,k\}) \geq 0, \ \forall j \in G \text{ and } \forall T \subseteq G \setminus \{j\} \\ G = \{\mathrm{EU},\mathrm{WU},\mathrm{GWP},\mathrm{ET},\mathrm{LO},\mathrm{AD}\} \end{array}\right\} \quad (26)$$

We have $\varepsilon^* = 0.136 > 0$, which implies that there exists at least one value function consistent with the preference information of the decision-maker. We then perform Step 6 to compute the necessary and possible preference relations in terms of each pair of fabrics. The results are summarized in Tables 5 and 6.

Table 5. Necessary preference relation in the set of fabrics

Fabric	Fabrics with necessary preferences ($\succsim^N$)
a_1	a_1
a_2	a_2
a_3	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{12}, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{24}, a_{25}, a_{26}$
a_4	$a_1, a_2, a_4, a_{17}, a_{18}, a_{21}, a_{24}, a_{25}$
a_5	$a_1, a_2, a_4, a_5, a_8, a_{12}, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{24}, a_{25}$
a_6	$a_1, a_2, a_4, a_6, a_{10}, a_{11}, a_{12}, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_7	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{12}, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{24}, a_{25}, a_{26}$
a_8	$a_1, a_2, a_4, a_5, a_8, a_{12}, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{24}, a_{25}$
a_9	$a_1, a_2, a_4, a_9, a_{12}, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{24}, a_{25}, a_{26}$
a_{10}	$a_1, a_2, a_{10}, a_{11}, a_{12}, a_{17}, a_{18}, a_{19}, a_{21}$
a_{11}	$a_1, a_2, a_{11}, a_{13}, a_{17}, a_{18}, a_{19}, a_{21}$
a_{12}	$a_1, a_2, a_{12}, a_{17}, a_{18}, a_{19}, a_{21}$
a_{13}	a_{13}, a_{18}
a_{14}	a_{14}, a_{15}, a_{16}
a_{15}	a_{15}
a_{16}	a_{16}
a_{17}	a_{17}, a_{18}, a_{19}
a_{18}	a_{18}
a_{19}	a_{19}
a_{20}	$a_1, a_2, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{24}, a_{25}$
a_{21}	a_{21}
a_{22}	$a_1, a_2, a_{13}, a_{17}, a_{18}, a_{19}, a_{22}$
a_{23}	$a_1, a_2, a_{13}, a_{17}, a_{18}, a_{19}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$

a_{24}	a_{21}, a_{24}
a_{25}	$a_1, a_2, a_{21}, a_{24}, a_{25}$
a_{26}	$a_1, a_2, a_4, a_9, a_{12}, a_{13}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{24}, a_{25}, a_{26}$

Notes: The fabric on the left column is necessarily preferred to the fabrics on the right. For example, in the last row, a_{26} is necessarily preferred to a_1 and a_2; that is, $a_{26} \succsim^N a_1$ and $a_{26} \succsim^N a_2$.

Table 6. Possible preference relation in the set of fabrics

Fabric	Fabrics with necessary preferences ($\succsim^P$)
a_1	$a_1, a_2, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}$
a_2	$a_1, a_2, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}$
a_3	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_4	$a_1, a_2, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_5	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_6	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_7	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_8	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_9	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{10}	$a_1, a_2, a_4, a_5, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{11}	$a_1, a_2, a_3, a_4, a_5, a_7, a_8, a_9, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{12}	$a_1, a_2, a_4, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{13}	$a_1, a_2, a_4, a_{10}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{14}	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{15}	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{16}	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{17}	$a_1, a_2, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{18}	$a_1, a_2, a_{14}, a_{15}, a_{16}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{19}	$a_1, a_2, a_4, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{20}	$a_1, a_2, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{21}	$a_1, a_2, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{22}$
a_{22}	$a_1, a_2, a_3, a_4, a_5, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{24}, a_{25}, a_{26}$
a_{23}	$a_1, a_2, a_3, a_4, a_5, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{24}	$a_1, a_2, a_{10}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{22}, a_{24}, a_{25}$
a_{25}	$a_1, a_2, a_{10}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{22}, a_{24}, a_{25}$
a_{26}	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$

Notes: The fabric on the left column is possibly preferred to the fabrics on the right. For example, in the last row, a_{26} is possibly preferred to a_1, a_2, a_4, etc.

From Tables 5 and 6, we observe that the fabrics 3 and 7 (a_3 and a_7) are the most preferred alternatives because they have the most number of necessary preferences relations as well as possible relations; on the contrary, the fabric 18 is the least preferred ones. That is, the fabrics 3 and 7 can be considered as a set of robust recommendations that is exactly consistent with the results in Gloria et al. (2013), where they chose fabrics 3 as the best alternative. We also observe several obvious preference reversals based on the learned necessary and possible relations, for example, a_{10} and a_{12} according to the results in Gloria et al. (2013). This preference reversal phenomenon exhibited in this study may be due to the merit of the preference learning model that considered the compromise effect. The previous method, however, cannot represent the compromise effect. In this regard, our learning model helps to better capture the decision-maker's unique compromise effect in decision process.

Fig. 2 displays the correlation matrices in terms of the learned multicriteria interaction from the *CCI*. In the graphs, x-axes and y-axes indicate the six criteria, all of which may be interactive to each other. Panel A of Fig. 2 indicates a prominent tendency for inter-dependent criteria towards positive interaction, and Panel B also exhibits notable positive interaction. However, Panel C of Fig. 2 shows that the correlation between pairs of criteria is relatively arbitrary, leading to a roughly even distribution of cell color on the spectrum. It is reasonable to expect that using the classical Choquet integral would lead to three identical graphs to Panel C, because the classical Choquet integral merely provides possible relations among fabrics 17, 18 and 19 (see next subsection and Appendix C).

In this sense, the apparent distortion of the first two images of Fig. 2 can be explained in the way that the *CCI* adds another layer of concavity on the aggregation process as explained in Section 3.2. The preference reversals are due to the concavity of the *CCI* that has proven to capture the decision-maker's unique preferences on intermediate alternatives. If carefully observing the data in Table 4, it is easy to find that a_{17} is an intermediate alternative in the local choice set constructed by its neighbors saying $S = \{a_{17}, a_{18}, a_{19}\}$. The evaluation values of a_{17} are located in the middle position of the criteria ET and LO (i.e., $0.2 < 0.3 > 0.4$, $0.6 < 0.7 > 0.8$). That is, the learned *CCI* function intensifies the utility of intermediate alternatives in the aggregation process and represents the positive interactions among criteria as shown in Panels A and B of Fig. 2.

5.3. Comparative analysis with the classical Choquet integral

In this section, we use the same real data for a fair comparison with the classical Choquet integral model to validate the performance of the *CCI*. First, we construct a *Chqouet integral* preference model based on the ROR technique and predict the preference relations. Second, we investigate the time complexity of the two methods using a randomly generated dataset.

We use the same dataset and learning setting as given in this section for the performance validation. The classical Choquet integral-based learning model, presented in Appendix A, is constructed by replacing the aggregation function in our learning approach with the Choquet integral operator. The resultant optimization problem is solved using the Matlab solver.[3] The results of the classical Choquet integral model in terms of necessary and possible relations are reported in Appendix B. One can observe in Tables B.1 and B.2 that the classical Choquet integral fails to offer results (*e.g.*, $a_{10} \succ^N a_{11}$) that our approach has provided. Fig. 3 shows two preference graphs learning from the classical Choquet integral and *CCI*, respectively. As the graphs shows, the Choquet integral-based model (panel A) does not obtain enough necessary preference relations to determine the most favorable alternative or the third one, while the proposed model (panel B) does. In other word, only our approach can succeed in learning the decision-maker's individualistic behavior which is the unique preferences on intermediate alternatives.

[3] https://www.mathworks.com/products/optimization

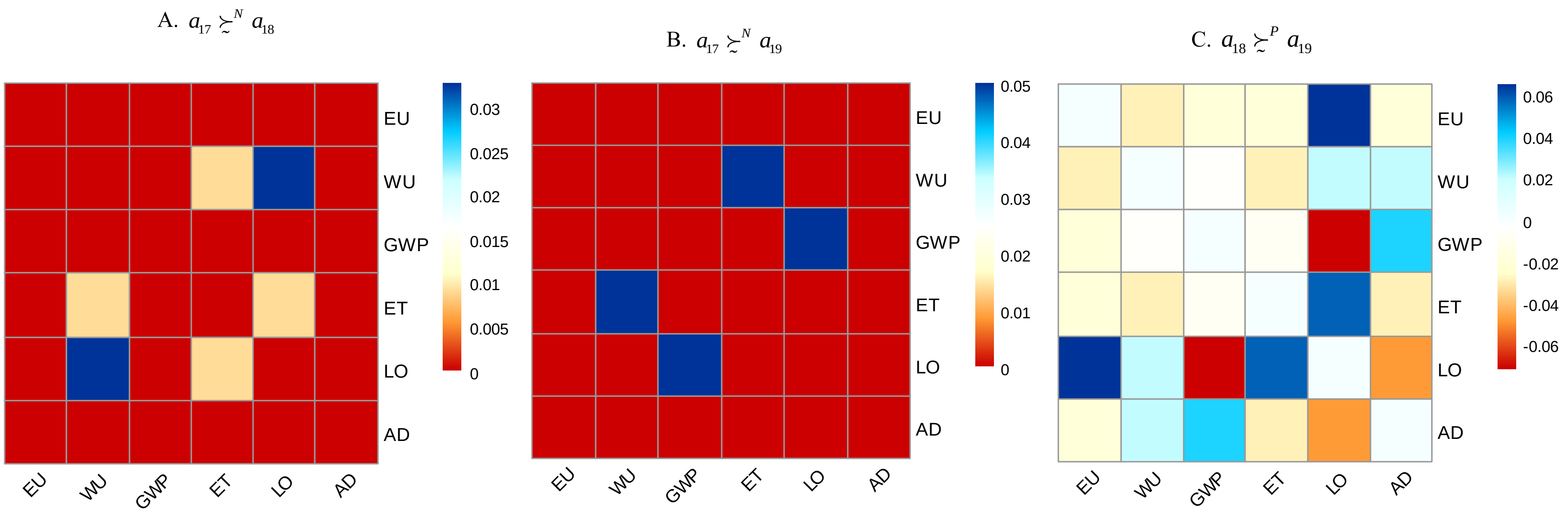
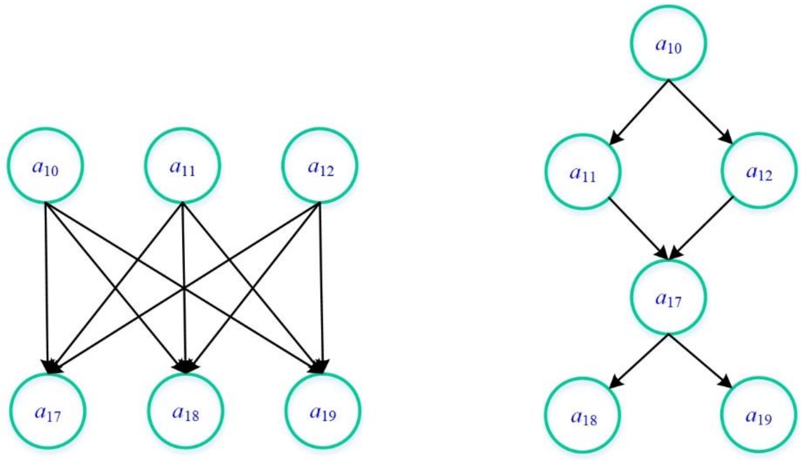


Fig. 2. Interaction heatmaps for three pairs of preference relations using *CCI* operator

Notes: The colored cell refers to the interaction index $I(\cdot)$. Panels A and B show the degrees of criteria interactions based on the preference relations $a_{17} \succsim^N a_{18}$ and $a_{17} \succsim^N a_{19}$, respectively, while Panel C reduces to the possible relation $a_{18} \succsim^P a_{19}$ where not all value functions are capable of reproduction of the preference relation.



A. classical Choquet integral B. *CCI*

Fig. 3. The *preference graphs* of necessary preference relations about six alternatives learned from the Choquet integral and *CCI*

To illustrate how the classical Choquet integral leads to different preferences, we employ the correlation maps and include the same pairs of alternatives used in Fig. 2 (see Appendix C). Panels A and B of Fig. C show that, compared with the *CCI*, the classical Choquet integral learns a set of interaction indices which are all but evenly distributed in the maps. This means that it is easy to learn a value function associated with the classical Choquet integral but not all. Accordingly, the classical Choquet integral implies that $a^{17} \succsim^P a^{18}$ and $a^{17} \succsim^P a^{19}$, whereas the *CCI* implies that $a^{17} \succsim^N a^{18}$ and $a^{17} \succsim^N a^{19}$. Generally, the classical Choquet integral does not represent the decision-maker's preferences on intermediate alternatives and not provide robust preference relations over the three alternatives.

5.4. Computational performance

To investigate timing comparisons versus the traditional Choquet integral model, we report the computational time of these two approaches while comparing with a baseline. As a baseline, the additive value function is applied in the comparative analysis because it is the root of other aggregation operators. We generate a set of input data points randomly, each of which represents the utility of a fabric. We estimate the model based on different datasets and predict the necessary preference relations by learning the instances. This prediction procedure was repeated 20 times. The reported runtime is shown in Tables 7 and 8, using a varying number of criteria and alternatives. The results are twofold. First, the *CCI* and the Choquet integral require more computational efforts than the additive value function. This is because, when considering the interactions between criteria, the optimization problem has additional $n(n-1)$ constraints on criteria interactions.

Table 7. Computational time of the proposed method and baseline methods with different numbers of alternatives

Number of alternatives	Method	Mean (s)	Min. (s)	Max. (s)
10	Type 1	2.0976	1.9336	2.3254
	Type 2	1.9946	1.9153	2.0820
	Type 3	1.6592	1.5935	1.7761
20	Type 1	9.5840	8.7808	11.3457
	Type 2	8.7890	8.2657	9.7466
	Type 3	7.9616	7.3726	9.0197
30	Type 1	20.5436	19.2431	22.3362
	Type 2	17.6535	16.0703	19.6985
	Type 3	15.4615	15.3078	15.7284
40	Type 1	29.3458	28.2901	32.9958
	Type 2	28.1947	27.5243	29.1599
	Type 3	27.7372	27.2911	28.2858

Note. Type 1: *CCI*; Type 2: Choquet integral; Type 3: additive value function

Table 8. Computational time of the proposed method and baseline methods with different numbers of criteria

Number of criteria	Method	Mean (s)	Min. (s)	Max. (s)
4	Type 1	12.6966	12.4419	13.1126
	Type 2	12.6489	12.3626	13.1137
	Type 3	12.2644	12.1116	12.5835
6	Type 1	12.2222	11.9815	12.7498
	Type 2	12.1729	11.8791	12.5693
	Type 3	11.8817	11.5133	12.3390
8	Type 1	12.9810	12.8426	13.4740
	Type 2	12.8816	12.7663	13.2476
	Type 3	12.2582	12.1612	12.6093
10	Type 1	13.0164	12.4197	14.8927
	Type 2	12.8430	12.1462	13.8447
	Type 3	12.2817	12.2105	13.9412

Note. Type 1: *CCI*; Type 2: Choquet integral; Type 3: additive value function

Second, the computing time of the learning model increases dramatically with the increase of alternatives, but it is not significantly affected by the number of criteria. The preference learning method is equivalent to solving an optimization problem where the constraints would increase $n(n-1)$ if n new alternatives are involved (see Models (24) and (25)). However, the time complexity will increase only if more criteria interaction information is given by decision-makers. In this case, the number of constraints in Model (22) would also increase quadratically.

6. Extensions

In this section, we present two extensions of our approach. We first discuss the situation where the interactivity process is involved which requires the decision-maker to specify the preferences incrementally. The second part extends the *CCI* to a data-driven construction to capture the decision-maker's preferences through a machine-learning technique.

6.1. Specifying preferences with an interactive process

The idea of specifying preferences by an interactive model with the decision-maker in further iterations is not a fresh one. In the context of MCDM, there are literature on improving the initial solutions through subsequent iterations. The early appearance of the interactivity approach was based on the idea that a decision-maker is asked to provide additional preference information throughout the solution process. This can be seen as looking for a feasible additive value function satisfied the necessary constraints. Alternatively, a few recent studies suggested an interactivity process that the algorithm asks for reactions from decision-makers so as to make further estimation of the parameters of value functions.

Similar to the process in Greco et al. (2013), when a necessary ranking is not enough to provide a final recommendation, we believe the decision model should accept additional constraints on the general value functions in order to explain the decision-maker's reaction in the next iteration. Practically speaking, in this interactive process, additional preferences can be identified as an interactivity with the decision-maker who gives feedbacks on the results of the last iteration. Note that the decision-maker is often unable to provide a set of proper preferences in previous iterations, yet s/he can give possibly small pieces after perceiving a lack of necessary relations.

Technically speaking, in each iteration, the additional pairwise comparisons can be treated as new constraints in the optimization problem shown as Model (23). When the new set of constraints $E_t^{S'}$ are satisfied in a particular iteration t, $t=\{1,2,\dots,l\}$, the model would narrow down the possible preference relations while enrich the necessary relations. Blow we propose an algorithm to extend our approach to implement the interactivity process.

If a decision-maker wishes to consider that the additional preference is narrowing down necessary rankings, then the constraints $E^{S'}$ presented in Model (23) can be extended to a new set of constraints by adding

$$CCI_{\mu,\lambda}(d)-CCI_{\mu,\lambda}(c)\le\varepsilon \tag{27}$$

where $(c,d)\in\mathcal{A}\times\mathcal{A}$ is a pair of alternatives. This piece of new preference information could lead to a significant change in the constraints (22). The resultant $E_t^{S'}$ needed for a non-empty polytope can be deemed as a narrowed set of constraints on the compatible value functions in the t th iteration. The partial preorders, including necessary and possible preference relations, can be derived easily by solving the optimization problems (24) and (25) after adding the new pairwise comparison. It is noticeable that the two preferences refer to the nested relations for the t th iteration such that $\succsim_{t-1}^{N}\subseteq\succsim_{t}^{N}$ and $\succsim_{t-1}^{P}\supseteq\succsim_{t}^{P}$, and the set of compatible value functions are also embedded in the inverse order diminished with the times of iterations such that $\mathcal{CCI}_1 \supseteq \ldots \supseteq \mathcal{CCI}_l$.

Considering the robust results in terms of two different relations, one can easily consider the interaction with the model to complement the necessary ranking and deteriorate the possible relations in order to provide a satisfactory recommendation. As a result, the interactivity approach lightens the decision-maker's burden of providing too much information at the first stage of decision and helps understand the impact of an individual's preference information on the learned preference relations. The proposed procedure allows the decision-maker to control the results in terms of necessary and possible preference relations, in which s/he is expected to provide more preference information to enrich the necessary ranking.

6.2. Machine learning

The *CCI* operator can also be extended to machine learning if a large amount of information about the decision-maker's preferences has been collected. In the previous numerical study, only a few pairwise comparisons are typically used as input data in the learning system. In the age of Internet, there often have hundreds of documents that come from the observation of decision-makers' preferences. Recent studies have presented a preference learning method using machine learning (Hüllermeier et al. 2008). Our proposed aggregation operator can also combine with machine learning techniques. In this subsection, instead of using the ROR method, we extend *CCI* to the machine learning context to predict the preferences of decision-makers.

We divide the data set into two parts, including training set $\mathcal{A}_{train}$ and testing set $\mathcal{A}_{test}$. For learning a mapping from pairwise comparison instances to ranking, the extended model first transfers pairwise preferences to learning labels. Specifically, we consider that the training set consists of pairwise preferences $\boldsymbol{a}_i \succ \boldsymbol{b}_i$ on $\mathcal{A}_{train} \times \mathcal{A}_{train}$. Each alternative $\boldsymbol{a} \in \mathcal{A}_{train}$ is characterized by n criteria, denoted by $G = \{1, 2, \ldots, n\}$. We consider that each alternative is related to a ground-truth ranking based on the decision-maker's preferences, denoted by a label ℓ_i where $\{\ell_1 > \ldots > \ell_m\}$. If the training data contain $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ associated with labels $\{\ell_1, \ell_2, \ldots, \ell_m\}$, a set of preference tuples can be constructed as $\mathcal{S} = \{(\boldsymbol{a}_i, \boldsymbol{b}_i) \mid \boldsymbol{a}_i \succ \boldsymbol{b}_i, 1 \le i \le L\}$ where L indicates the number of pairwise comparisons in the training set $\mathcal{A}_{train}$. The task is to learn the optimal values of parameters in $CCI_{\mu,\lambda}$ for representing the preference information in $\mathcal{S}$.

In what follows, we describe an optimization problem that maximizes the margin of misranking errors.

$$\max \varepsilon_i \tag{28}$$

$$s.t. \begin{cases} CCI_{\mu,\lambda}(\boldsymbol{a}_i) \geq CCI_{\mu,\lambda}(\boldsymbol{b}_i) + \varepsilon_i \\ \varepsilon_i \geq 0,\ \forall i \in \{1,2,\ldots,L\} \\ m^{\mu}(\varnothing) = 0 \\ \sum_{j \in C} m^{\mu}(\{j\}) + \sum_{\{j,k\} \subseteq G} m^{\mu}(\{j,k\}) = 1 \\ m^{\mu}(\{j\}) \geq 0 \\ m^{\mu}(j) + \sum_{k \in T} m^{\mu}(\{j,k\}) \geq 0, \forall j \in G \text{ and } \forall T \subseteq G \setminus \{j\} \end{cases}$$

The objective of Model (28) is to maximize the slack variable ε_i which is related to each pair of preference tuple $(\boldsymbol{a}_i, \boldsymbol{b}_i)$. In a recent study, Aggarwal and Fallah Tehrani (2019) proposed a version of preference learning used in kernel-based machine-learning methods. Specifically, the objective function in Model (28) is replaced by

$$\max_{\mathcal{M},\varepsilon_1,\ldots,\varepsilon_L} \{\mathcal{M} - \frac{\gamma}{|\mathcal{S}|} \sum_{(\boldsymbol{a}_i,\boldsymbol{b}_i) \in \mathcal{S}} (\varepsilon_i^{\boldsymbol{a}} + \varepsilon_i^{\boldsymbol{b}})\} \tag{29}$$

where $\mathcal{M}$ indicates the margin to maximize the smallest utility difference for a pairwise preference $\boldsymbol{a}_i \succ \boldsymbol{b}_i$. The slack variables $\varepsilon_i^{\boldsymbol{a}}$ and $\varepsilon_i^{\boldsymbol{b}}$ correspond to alternatives $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ for a preference tuple $(\boldsymbol{a}_i, \boldsymbol{b}_i)$. γ is a trade-off parameter that ensures the flexibility of Model (28). Consequently, the constraint $CCI_{\mu,\lambda}(\boldsymbol{a}_i) \geq CCI_{\mu,\lambda}(\boldsymbol{b}_i) + \varepsilon_i$ in Model (28) can be replaced by a new inequality:

$$CCI_{\mu,\lambda}(\boldsymbol{a}_i) - CCI_{\mu,\lambda}(\boldsymbol{b}_i) > \mathcal{M} - \varepsilon_i^{\boldsymbol{a}} - \varepsilon_i^{\boldsymbol{b}} \qquad \forall (\boldsymbol{a}_i, \boldsymbol{b}_i) \in \mathcal{S} \tag{30}$$

Unlike our preference model, the extended machine learning model only makes use of those alternatives that have corresponding labels. In this sense, machine learning techniques are different from the multiple criteria analysis which belongs to decision support tools. We note that the preference learning in machine learning requires massive preference information as input data that may be difficult to obtain in practice. In addition, machine learning models usually have a tuning hyperparameter, i.e., γ in Model (29), which may lead to extra time complexity in the learning process.

7. Conclusions

This paper advances both the theory and practice of preference learning. In particular, the paper developed a Choquet integral-based preference learning model that can be used to support decision making in view of multiple interactive criteria and human's irrational choice. The theoretical findings show that if multiple criteria are not independent to each other and decision-makers' choices are affected by the other alternatives, then these preferences can be represented by a concave aggregation function. This paper also developed an optimization model to identify the whole set of value functions compatible with the preference information. Moreover, the paper showed that preference learning techniques, differently from the conventional preference learning, can be used to find robust decision recommendations based on preference statements given by decision-makers. The application of the developed preference learning model to the real sustainable supply chain data set showed that the proposed approach can provide a set of robust decision recommendations that are compatible with decision-makers' preferences. The comparison analysis suggested that the proposed approach outperforms the traditional Choquet integral model in both accuracy and computing aspects.

There are still research questions that are beyond the scope of this study. The main bottleneck of the optimization problem in Model (23) is that the last inequation contains $n(n-1)$ constraints. The computational complexity of the model might cause difficulties in practical applications. There are several avenues for future research. First, empirical studies on how the changes of the concavity parameter λ may affect the preference of individual alternative could help identify those approaches for representing the varying compromise effect that we have not considered. Second, we can see some prospects for preference learning in the rough set area, using the dominance-based rough set as suggested by Wallenius et al. (2008). Finally, helping organizations incorporate sustainability into marketing strategies and production decisions would be a promising application for further studies.

Acknowledgments The work was supported by the National Natural Science Foundation of China (72171158, 71971145, 71771156).

References

Aggarwal, M., & Fallah Tehrani, A. (2019). Modelling human decision behaviour with preference learning. *INFORMS Journal on Computing*, *31*(2), 318-334.

Angilella, S., Corrente, S., Greco, S., & Słowiński, R. (2016). Robust Ordinal Regression and Stochastic Multiobjective Acceptability Analysis in multiple criteria hierarchy process for the Choquet integral preference model. *Omega*, *63*, 154-169.

Arcidiacono, S. G., Corrente, S., & Greco, S. (2020). As simple as possible but not simpler in Multiple Criteria Decision Aiding: the robust-stochastic level dependent Choquet integral approach. *European Journal of Operational Research*, *280*(3), 988-1007.

Bertsimas, D., & O'Hair, A. (2013). Learning preferences under noise and loss aversion: An optimization approach. *Operations Research*, *61*(5), 1190-1199.

Bouaziz, M. F., Marange, P., Voisin, A., & Petin, J. F. (2021). Control Sequence Ranking for Critical System Based on Health of Equipment Thanks to Choquet Integral. *Frontiers in Artificial Intelligence*, *3*, 614853.

Carli, R., Dotoli, M., & Pellegrino, R. (2018). Multi-criteria decision-making for sustainable metropolitan cities assessment. *Journal of environmental management*, *226*, 46-61.

Choquet, G. (1954). Theory of capacities. In *Annales de l'institut Fourier* (Vol. 5, pp. 131-295).

Cobuloglu, H. I., & Büyüktahtakın, İ. E. (2015). A stochastic multi-criteria decision analysis for sustainable biomass crop selection. *Expert Systems with Applications*, *42*(15-16), 6065-6074.

Corrente, S., Greco, S., Kadziński, M., & Słowiński, R. (2013). Robust ordinal regression in preference learning and ranking. *Machine Learning*, *93*(2), 381-422.

Corrente, S., Greco, S., & Ishizaka, A. (2016). Combining analytical hierarchy process and Choquet integral within non-additive robust ordinal regression. *Omega*, *61*, 2-18.

Dhurkari, R. K. (2022). MCDM methods: Practical difficulties and future directions for improvement. *RAIRO-Operations Research*, *56*(4), 2221-2233.

Dyer, J. S., Fishburn, P. C., Steuer, R. E., Wallenius, J., & Zionts, S. (1992). Multiple criteria decision making, multiattribute utility theory: the next ten years. *Management science*, *38*(5), 645-654.

Erol, İ., Sencer, S., Özmen, A., & Searcy, C. (2014). Fuzzy MCDM framework for locating a nuclear power plant in Turkey. *Energy Policy*, *67*, 186-197.

Gloria, T. P., Kohlsaat, C., Bautil, P., Wolf, B., Early, D., & Ben-Zekry, B. (2014). A statistical approach to interpret relative environmental performance within product categories. *The International Journal of Life Cycle Assessment*, *19*(3), 491-499.

Grabisch, M. (1996). The application of fuzzy integrals in multicriteria decision making. *European journal of operational research*, *89*(3), 445-456.

Grabisch, M. (1997a). Alternative representations of OWA operators. In *The Ordered Weighted Averaging Operators* (pp. 73-85). Springer, Boston, MA.

Grabisch, M. (1997b). K-order additive discrete fuzzy measures and their representation. *Fuzzy sets and systems*, *92*(2), 167-189.

Greco, S., Figueira, J., & Ehrgott, M. (2016). *Multiple criteria decision analysis* (Vol. 37). New York: springer.

Greco, S., Mousseau, V., & Słowiński, R. (2008). Ordinal regression revisited: multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research*, *191*(2), 416-436.

Greco, S., Mousseau, V., & Słowiński, R. (2010). Multiple criteria sorting with a set of additive value functions. *European Journal of Operational Research*, *207*(3), 1455-1470.

Greco, S., Mousseau, V., & Słowiński, R. (2014). Robust ordinal regression for value functions handling interacting criteria. *European Journal of Operational Research*, *239*(3), 711-730.

Guo, L. (2016). Contextual deliberation and preference construction. *Management Science*, *62*(10), 2977-2993.

Howes, A., Warren, P. A., Farmer, G., El-Deredy, W., & Lewis, R. L. (2016). Why contextual preference reversals maximize expected value. *Psychological review*, *123*(4), 368.

Hüllermeier, E., Fürnkranz, J., Cheng, W., & Brinker, K. (2008). Label ranking by learning pairwise preferences. *Artificial Intelligence*, *172*(16-17), 1897-1916.

Keeney, R. L., Raiffa, H., & Meyer, R. F. (1993). *Decisions with multiple objectives: preferences and value trade-offs*. Cambridge university press.

Kivetz, R., Netzer, O., & Srinivasan, V. (2004). Alternative models for capturing the compromise effect. *Journal of marketing research*, *41*(3), 237-257.

Labreuche, C., & Grabisch, M. (2006). Generalized Choquet-like aggregation functions for handling bipolar scales. *European Journal of Operational Research*, *172*(3), 931-955.

Liao, Z. Q., Liao, H. C., Tang, M., Al-Barakati, A., & Llopis-Albert, C. (2020). A Choquet integral-based hesitant fuzzy gained and lost dominance score method for multi-criteria group decision making considering the risk preferences of experts: Case study of higher business education evaluation. *Information Fusion*, *62*, 121-133.

Mastrocinque, E., Ramírez, F. J., Honrubia-Escribano, A., & Pham, D. T. (2020). An AHP-based multi-criteria model for sustainable supply chain development in the renewable energy sector. *Expert Systems with Applications*, *150*, 113321.

Mayag, B., & Bouyssou, D. (2020). Necessary and possible interaction between criteria in a 2-additive Choquet integral model. *European Journal of Operational Research*, *283*(1), 308-320.

Murofushi, T., & Soneda, S. (1993). Techniques for reading fuzzy measures (III): interaction index. In *9th fuzzy system symposium* (pp. 693-696).

Reimann, O., Schumacher, C., & Vetschera, R. (2017). How well does the OWA operator represent real preferences?. *European journal of operational research*, *258*(3), 993-1003.

Rota, G. C. (1964). On the foundations of combinatorial theory I. Theory of Möbius functions. *Probability Theory and Related Fields*, *2*(4), 340-368.

Sánchez-Garrido, A. J., Navarro, I. J., & Yepes, V. (2022). Multi-criteria decision-making applied to the sustainability of building structures based on Modern Methods of Construction. *Journal of Cleaner Production*, *330*, 129724.

Shapley, L. S. (1953). A value for n-person games. *Annals of Mathematics Studies*, *28*, 307-317.

Singh, A., & Kumar, S. (2021). Picture fuzzy Choquet integral-based VIKOR for multicriteria group decision-making problems. *Granular Computing*, *6*(3), 587-601.

Simonson, I., & Tversky, A. (1992). Choice in context: Tradeoff contrast and extremeness aversion. *Journal of marketing research*, *29*(3), 281-295.

Sugeno, M. (1974). Theory of fuzzy integrals and its applications. *Doct. Thesis, Tokyo Institute of technology*.

Tehrani, A. F., Cheng, W., & Hullermeier, E. (2012). Preference learning using the Choquet integral: The case of multipartite ranking. *IEEE Transactions on Fuzzy Systems*, *20*(6), 1102-1113.

Tversky, A., & Simonson, I. (1993). Context-dependent preferences. *Management science*, *39*(10), 1179-1189.

Wallenius, J., Dyer, J. S., Fishburn, P. C., Steuer, R. E., Zionts, S., & Deb, K. (2008). Multiple criteria decision making, multiattribute utility theory: Recent accomplishments and what lies ahead. *Management science*, *54*(7), 1336-1349.

Yager, R. R. (1998). Including importances in OWA aggregations using fuzzy systems modeling. *IEEE Transactions on Fuzzy Systems*, *6*(2), 286-294.

Appendix A. Classical Choquet integral-based preference model

This approach suggests an aggregation process that is characterized by the underlying fuzzy measure $\mu(\cdot)$ for capturing the interaction among criteria. Our learning task becomes simple to determine $\mu(\cdot)$. To do so, we express the Choquet integral using the Möbius transform shown as Eq. (4), constituting an identical representation of fuzzy measure, and then we learn the parameter of $m(T)$, $\forall T \subseteq C$. We determine the parameter by minimizing the misranking error of the preference tuples $\boldsymbol{a} \succsim \boldsymbol{b}$. We seek to maximize the margin of misranking error, typically used in the ROR, and our preference learning task becomes a linear program:

$$\varepsilon_N^* = \max \varepsilon$$
$$s.t.\ \begin{cases} C_\mu(\boldsymbol{b}^*) \geq C_\mu(\boldsymbol{a}^*) + \varepsilon \\ E_c^{S'} \end{cases}$$
$$\varepsilon_P^* = \max \varepsilon$$
$$s.t.\ \begin{cases} C_\mu(\boldsymbol{a}^*) \geq C_\mu(\boldsymbol{b}^*) \\ E_c^{S'} \end{cases}$$

where $E_c^{S'}$ is a set of constraints that only replace the aggregation function $CCI_{\mu,\lambda}$ in $E^{S'}$ with the Choquet integral operator C_μ. The classical Choquet integral only considers the interaction effect among criteria but ignore the compromise effect. For a comparative purpose, the Choquet integral provides a natural baseline to illustrate the ability of the *CCI* operator for modeling the decision-makers' compromise effect.

Appendix B. The solved preference relations based the classical Choquet integral model

Table B.1. Necessary preference relation in the set of fabrics

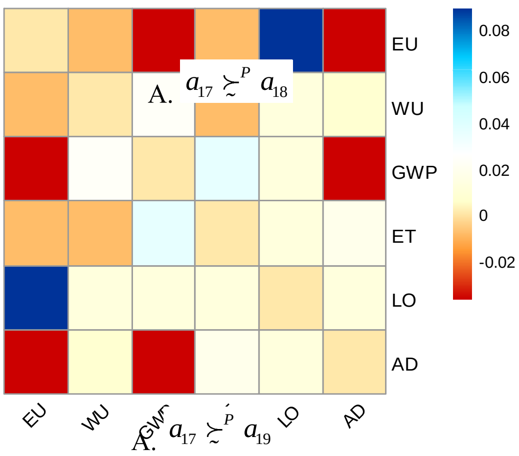
Fabric	Fabrics with necessary preferences ($\succsim^N$)
a_1	a_1
a_2	a_2
a_3	$a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25},a_{26}$
a_4	$a_1,a_2,a_4,a_{13},a_{17},a_{18},a_{19},a_{21},a_{24},a_{25}$
a_5	$a_1,a_2,a_4,a_5,a_8,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25}$
a_6	$a_1,a_2,a_4,a_6,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25},a_{26}$
a_7	$a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25},a_{26}$
a_8	$a_1,a_2,a_4,a_5,a_8,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25}$
a_9	$a_1,a_2,a_4,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25},a_{26}$
a_{10}	$a_1,a_2,a_{10},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{21}$
a_{11}	$a_1,a_2,a_{11},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{21}$
a_{12}	$a_1,a_2,a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{21}$
a_{13}	a_{13},a_{18}
a_{14}	a_{14},a_{15},a_{16}
a_{15}	a_{15}
a_{16}	a_{16}
a_{17}	a_{17}
a_{18}	a_{18}
a_{19}	a_{19}
a_{20}	$a_1,a_2,a_4,a_{13},a_{17},a_{18},a_{19},a_{20},a_{21},a_{24},a_{25}$
a_{21}	a_{21}
a_{22}	$a_1,a_2,a_{12},a_{13},a_{17},a_{18},a_{19},a_{21},a_{22}$
a_{23}	$a_1,a_2,a_{13},a_{17},a_{18},a_{19},a_{21},a_{22},a_{23},a_{24},a_{25}$
a_{24}	a_{24}
a_{25}	$a_1,a_2,a_{17},a_{18},a_{19},a_{21},a_{24},a_{25}$
a_{26}	$a_1,a_2,a_4,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25},a_{26}$

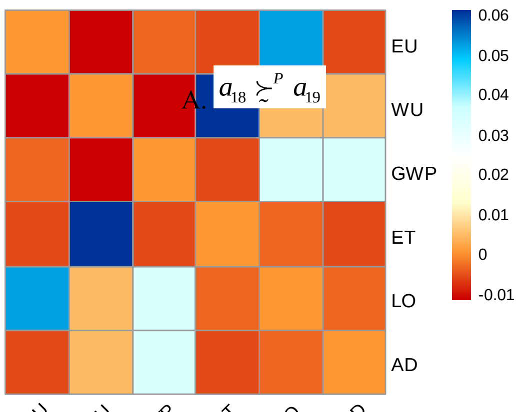
Table B.2. Possible preference relation in the set of fabrics

Fabric	Fabrics with necessary preferences ($\succsim^P$)
a_1	$a_1,a_2,a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{21},a_{24},a_{25}$
a_2	$a_1,a_2,a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{21},a_{24},a_{25}$
a_3	$a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{20},a_{21},a_{22},a_{23},a_{24},a_{25},a_{26}$
a_4	$a_1,a_2,a_4,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19},a_{21},a_{22},a_{23},a_{24},a_{25}$

a_5	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_6	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_7	$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_8	$a_1, a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_9	$a_1, a_2, a_4, a_5, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$
a_{10}	$a_1, a_2, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{11}	$a_1, a_2, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{12}	$a_1, a_2, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{13}	$a_1, a_2, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{14}	$a_1, a_2, a_4, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{15}	$a_1, a_2, a_4, a_{13}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{16}	$a_1, a_2, a_4, a_{13}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{17}	$a_1, a_2, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{18}	$a_1, a_2, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{19}	$a_1, a_2, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{24}, a_{25}$
a_{20}	$a_1, a_2, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{21}	$a_1, a_2, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}$
a_{22}	$a_1, a_2, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{24}, a_{25}$
a_{23}	$a_1, a_2, a_3, a_4, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}$
a_{24}	$a_1, a_2, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{22}, a_{24}$
a_{25}	$a_1, a_2, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{21}, a_{22}, a_{24}, a_{25}$
a_{26}	$a_1, a_2, a_4, a_5, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}$

Appendix C. The correlation matrices based on the classical Choquet integral model





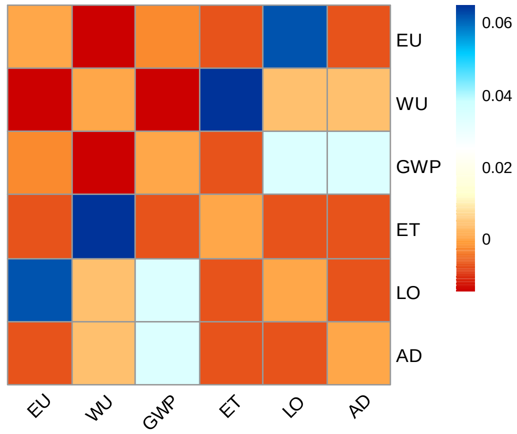


Fig. C. Interaction heatmaps for three pairs of preference relations base on the classical Choquet integral